10 Inflation

- So far we have a pretty good understanding of the Universe, based on quantitative agreement between models and observations. But, the Hot Big Bang model where the Universe is initially radiation dominated has problems. Now we go back even further in time to consider an idea that solves them, called "inflation".

First, we take a look at the problems.

The Flatness Problem

- We know from equations (4.13) and (4.14) that the curvature of the Universe cannot change from open to closed, or vice versa, and that a perfectly flat Universe is always flat. However, a small deviation from flatness very early on is amplified, so requiring $\kappa = 0$ as some kind of cosmic coincidence is unsatisfactory (a "fine tuning" problem).

- To consider the time evolution of Ω, we take equation (4.13) divided by (4.14).

$$1 - \Omega(t) = \frac{H_0^2 (1 - \Omega_0)}{H(t)^2 a(t)^2} \qquad (10.1)$$

if we consider times $t \ll t_{mr}$ and a near-flat universe, we can use equation (5.21):

$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} \qquad (10.2)$$

then substituting we get

$$1-\Omega(t) = \frac{(1-\Omega_0)\,a(t)^2}{\Omega_{r,0} + a(t)\Omega_{m,0}} \qquad (10.3)$$

so when radiation dominates (remembering $a \propto t^{1/2}$)

$$|1-\Omega|_r \propto a^2 \propto t \qquad (10.4)$$

and when matter dominates (remembering $a \propto t^{2/3}$)

$$|1-\Omega|_m \propto a \propto t^{2/3} \qquad (10.5)$$

so if $\Omega \neq 1$, it grows away from 1 over the history of the Universe.

- How much? using the Benchmark model ($\Omega_{r,0} \approx 10^{-4}$, $\Omega_{m,0} \approx 0.31$) and with $|1-\Omega_0| \leq 0.005$

  - at t_{rm}, $a_{rm} = 2.9 \times 10^{-4}$ so $|1-\Omega|_{rm} \leq 1.5 \times 10^{-6}$

  - at nucleosynthesis $a_{nuc} \approx 3.6 \times 10^{-9}$, so

$$|1-\Omega|_{nuc} \leq 2 \times 10^{-16}$$

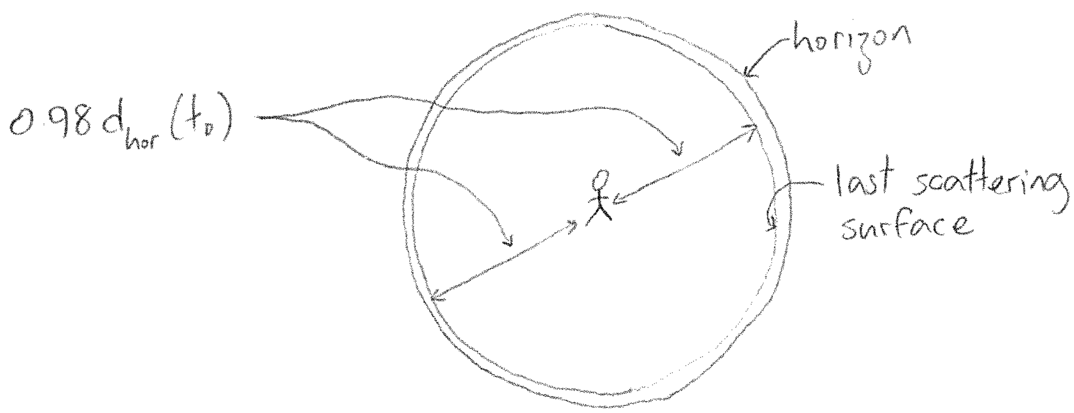
  - at the Planck time (5×10^{-44} s) $a_p \approx 2 \times 10^{-32}$, so

$$|1-\Omega|_p \leq 7 \times 10^{-63}$$

- It is fine tuning in the extreme to require Ω_p to be this small by coincidence, so we should seek a physical explanation.

The Horizon Problem

- This problem arises from the observation that the CMB has a near-uniform temperature, wherever we look. The proper distance to the last scattering surface is only a bit smaller than the horizon distance. Thus two opposite points on the sky are separated by $\approx 2d_{hor}$, and cannot be causally connected (so have not had time to come in to thermal equilibrium).



formally the proper distance to the horizon is

$$d_{hor}(t_0) = c \int_0^{t_0} \frac{dt}{a(t)} \qquad (10.6)$$

and to the last scattering surface is

$$d_{ls}(t_0) = c \int_{t_{ls}}^{t_0} \frac{dt}{a(t)} \qquad (10.7)$$

- In fact, the size of the horizon distance at the time of last scattering is only $\sim 1^\circ$ on the sky, so we need not use opposite sides of the sky to find regions that aren't causally connected.

The Monopole Problem

- We have never observed magnetic monopoles, or other particles that may be a relic of the Big Bang.

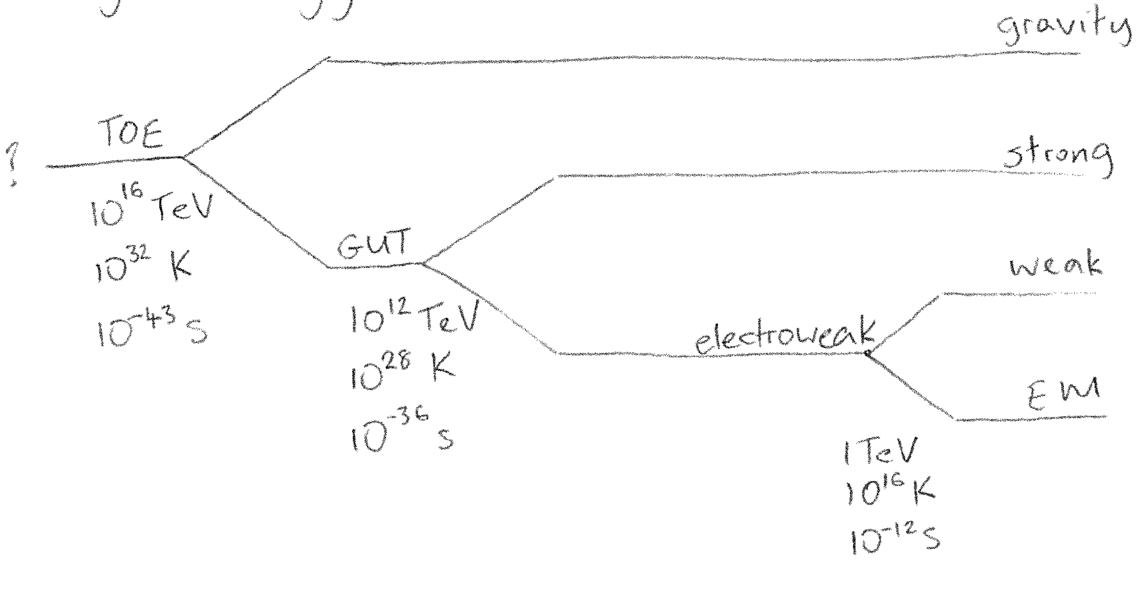
- These are predicted by "Grand Unification Theories", so we will take a quick look at what they are. Unification theories aim to explain multiple forces with a single theory (e.g. electric and magnetic forces → Maxwell's electromagnetism).

  The oveall goal is to explain gravity, the strong and weak forces, and electromagnetism with one Theory of Everything, but we need not do this all at once, as below certain energies these forces appear very different.

  - the "electroweak" force unifies the weak and electromagnetic forces, by showing that energies above about 1 TeV, the W and Z bosons (weak force) and the photon combine into electroweak bosons (W_1, W_2, W_3, B).

  - a GUT would combine the electroweak and strong forces, at an even higher energy ($\sim 10^{12}$ TeV).

  - a TOE would combine a GUT with gravity, at a yet higher energy ($\sim 10^{16}$ TeV).



- we expect that at the earliest times in the Universe, all four forces were unified, but that they become distinct (called "symmetry breaking") as the Universe cooled, and went through "phase transitions".

- these phase transitions can give rise to "topological defects" which are point-like and act as magnetic monopoles. If they exist we should see them, but we don't, implying that their number density is very low.

The Inflation Solution

- Alan Guth proposed the idea of "inflation", a period in the early Universe when the scale factor is accelerating, as a solution to all these problems. The acceleration equation (4.25) tells us that $\ddot{a} > 0$ requires $P < -\varepsilon/3$, so a cosmological constant Λ_i can be used to model the behaviour.

- Consider an inflationary phase when the energy density is dominated by a cosmological constant. The Friedmann equation is (noting that H_i is constant)

$$H_i^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda_i}{3} \qquad (10.8)$$

with solution for the scale factor evolution (i.e. equation 5.14)

$$a(t) \propto e^{H_i t} \qquad (10.9)$$

- Now, by assuming that there was a period of inflation from t_i to t_f, we can solve the aforementioned problems. In this time the scale factor increases by e^N, where N is the number of Hubble times H_i^{-1}.

- From equation (4.13), removing κ by using the abolute value,

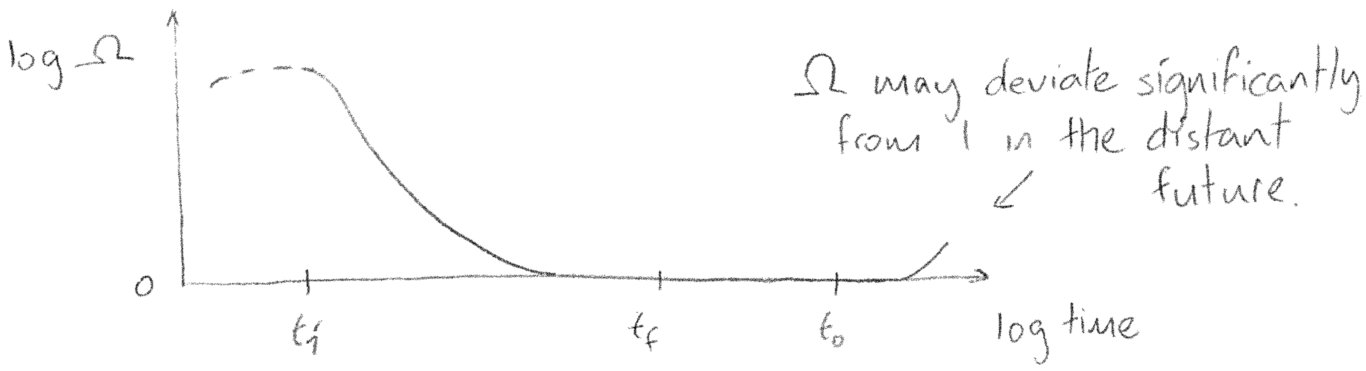
$$\left|1 - \Omega(t)\right| = \frac{c^2}{R_0^2\, a(t)^2 H_i^2} \tag{10.10}$$

and substituting $a \propto e^{H_i t}$, we get

$$\left|1 - \Omega(t)\right| \propto e^{-2H_i t} \tag{10.11}$$

So the difference between 1 and Ω plummets with time during inflation exponentially, and a sufficiently long period of inflation will make $1 - \Omega \approx 0$ and solve the flatness problem.

- for example, if inflation occurred at the GUT symmetry breaking $t \sim 10^{-36}$ s, then flatness today requires $\left|1 - \Omega(t_f)\right| \lesssim 10^{-52}$, and thus an inflationary period that lasted at least 60 Hubble times. Since the Hubble time is $H_i^{-1} \approx t_{GUT}$, inflation in this case would need to last at least $\sim 10^{-34}$ s.

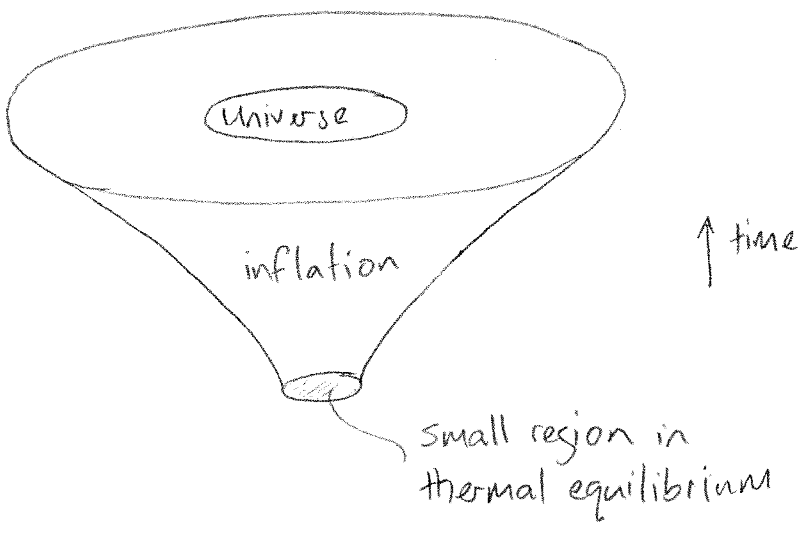


- The horizon problem is solved because a small patch of Universe that came to thermal equilibrium before inflation is then expanded to a region larger than our horizon distance.

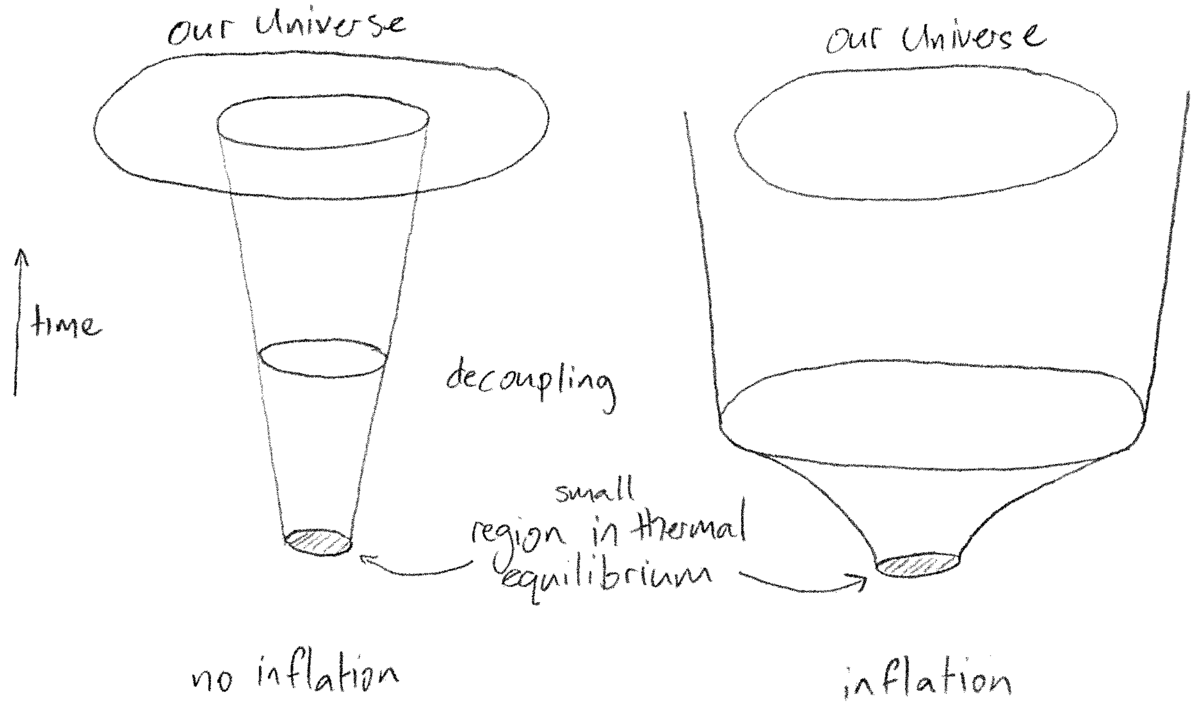
  - before inflation the horizon distance was roughly $ct_i = 3 \times 10^{-28}$ m. If we assume 65 Hubble times of expansion, then afterwards it was $e^{65} ct_i = 5$m. So after inflation pieces of the Universe closer than about 5m were causally connected, and could have been in thermal equilibrium.

  - the scale factor at the end of inflation (which can be estimated from equation (10.3) was $a(t_f) \sim 2 \times 10^{-27}$. Our current horizon distance is roughly $c/H_0 = 4400$ Mpc. Thus, at the end of inflation our currently visible Universe was stuffed into a ball about 60cm in diameter.

  - thus our currently visible Universe is expected to have been in thermal equilibrium, and thus the CMB appear the same everywhere on the sky.



- Another way of thinking of how inflation solves the horizon problem:
  - Normally, we expect that light travels a much greater distance after decoupling than before, so our observable Universe is larger than causally connected regions at decoupling.
  - With inflation, light travels much farther between the Big Bang and decoupling than after decoupling, so causally connected regions are greater than our observable Universe.



- The monopole problem is solved by the exponential growth of the scale factor, which strongly decreases their number density.

What causes inflation?

Thought to be driven by some as-yet undiscovered matter/particle that exists at very high energies (early times).