12 Structure Formation

- Thus far we have asserted that the Universe is homogeneous and isotropic, which allowed us to create models of how the Universe as a whole evolves.

  Clearly, on scales smaller than 100Mpc this assertion fails because we observe galaxies, clusters, stars, etc. We also see fluctuations in the CMB temperature, which indicates that there were slight inhomogeneities in density at the time of last scattering.

  - the growth of small density perturbations into "large scale structure" (eg. 2dF GRS structure) can be explained by "gravitational instability" (see also Matthew 13:12).

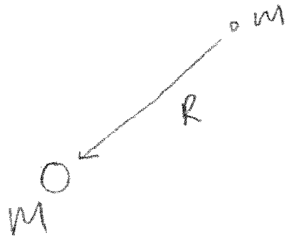
Gravitational instability

- Also known as "Jeans instability", this is the collapse of an initial overdensity by gravity. A density perturbation is not guaranteed to collapse, as it may be supported by pressure (in which case we have hydrostatic equilibrium, Earth's atmosphere for example). In the absence of sufficient pressure, density perturbations grow exponentially with time.

  - thus two timescales are relevant; the collapse time, and the time it would take the pressure in the medium to react to the resultant density increase. Pressure changes at the speed of sound, so the timescale is how long it takes a sound wave to traverse the region that is collapsing.

- by equating these timescales, we can find the threshold size, known as the "Jeans length", <u>above</u> which overdense regions will collapse under their own gravity.

- The "collapse time", also known as the "freefall time" and the "dynamical time", can be estimated by considering how long it takes a particle to fall on to a mass from some distance.



$$a = \frac{GM}{R^2} \quad , \text{ and } \quad R = \frac{1}{2} a t^2$$

using the mean density $M = \frac{4}{3}\pi R^3 \bar{\rho}$, as by Gauss' shell theorem we could equally be considering a sphere of mass M and radius R, then (ignoring prefactors)

$$t_{dyn} \sim \frac{1}{\sqrt{G\bar{\rho}}} = \left(\frac{c^2}{G\bar{\varepsilon}}\right)^{1/2} \qquad (12.1)$$

- The speed of sound depends on the medium, but we can use our simple equation of state (4.28) $P = w\varepsilon$. We then have

$$c_s = \left(\frac{\partial P}{\partial \rho}\right)^{1/2} = c\left(\frac{dP}{d\varepsilon}\right)^{1/2} = \sqrt{w}\, c \qquad (12.2)$$

and the sound-crossing time is approximately

$$t_{pre} = \frac{R}{c_s} \qquad (12.3)$$

- For a region to collapse, we must have

$$t_{dyn} < t_{pre} \tag{12.4}$$

and the size of the region when they are equal (i.e. the critical size above which the region will collapse) is

$$\lambda_J \sim c_s t_{dyn} = c_s \left(\frac{c^2}{G\bar{\varepsilon}} \right)^{1/2} \tag{12.5}$$

(a more quantitative derivation has an extra $\sqrt{\pi}$ on the RHS)

we can also define a Jeans mass,

$$M_J = \frac{4}{3} \pi \bar{\rho} \lambda_J^3$$

Jeans instability in radiation and matter dominated universes

- We of course need to compare the Jeans length to physical scales, such as the known sizes of galaxies and clusters, and to the size of our observable Universe.

  Assuming a flat (i.e. critical density) universe, the characteristic expansion time is the Hubble time

$$H^{-1} = \left(\frac{3c^2}{8\pi G \bar{\varepsilon}} \right)^{1/2} \tag{12.6}$$

aside from the numerical factors, equations (12.1) and (12.6) are the same.

therefore, rewriting the Jeans length from equation (12.5)

$$\lambda_J \sim c_s t_{dyn} \sim \frac{c_s}{H} = \sqrt{w}\,\frac{c}{H} = \sqrt{w}\, d_{hor} \qquad 12.7$$

- in a radiation dominated universe $w = 1/3$, so the Jeans length is comparable to the horizon distance. The Jeans masses are much larger than supercluster masses, so the structures we observe must have formed later.

- in a matter dominated universe $w = 0$; clearly this does not help desive a Jeans length.

- what we should be considering is the sound speed in the baryon gas just after the decoupling of photons and baryons (see section 8 on CMB). What we'll consider is the sudden drop in Jeans length for the baryons at decoupling, caused by a drop in their sound speed.

  just before decoupling the Jeans length was

$$\lambda_J(\text{before}) \sim \sqrt{w}\,\frac{c}{H} \sim 0.04\ \text{Mpc} \qquad (12.8)$$

and just after, for the baryons it drops by a factor $F = c_s(\text{baryon})/c_s(\text{photon})$.

$$c_s(\text{photon}) = \frac{c}{\sqrt{3}} \simeq 0.58\,c \qquad (12.9)$$

$$c_s(\text{baryon}) = \left(\frac{k_B T}{m c^2}\right)^{1/2} c \qquad (12.10)$$

(from equation (4.30), $P = \frac{k_B T}{mc^2}\,\varepsilon$).

at decoupling $T_{dec} = 2970\,K$, so $k_B T_{dec} = 0.26\,eV$. If we assume $Y_p = 0.24$, then the mean rest mass is $1.22\, m_p c^2 = 1145\, MeV$.

$$c_s(\text{baryon}) = \left(\frac{0.26}{1145 \times 10^6} \right)^{1/2} = 1.5 \times 10^{-5}\, c \qquad (12\text{-}11)$$

therefore $F \approx 2.6 \times 10^{-5}$, and the Jeans length for baryons drops sharply at decoupling, so structures can form.

- This description is highly simplified. Aside from ignoring various pre-factors, the Universe is expanding, which counters the density increases as regions start to collapse. A proper treatment of structure formation by gravitational instability must therefore consider collapse and expansion self consistently.