6 Measuring Cosmological Parameters

- So far we know that the redshift of a distant galaxy tells us the scale factor at the time the light was emitted (relative to $a = 1$ now at t_0, equation (2-15)). This doesn't however tell us how the scale factor evolves.

It is possible to constrain $a(t)$, and since $a(t)$ depends on the energy density of the components that make up the Universe, we can set constraints on ε for matter and Λ by observing objects out to $z \sim 1$.

Deceleration parameter

- Consider a Taylor series expansion for $a(t)$ around the present time.

$$a(t) = a(t_0) + \left.\frac{da}{dt}\right|_{t_0} (t - t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t_0} (t - t_0)^2 + \cdots \qquad (6\text{-}1)$$

The scale factor variations derived above are smoothly varying, so we expect that this series is a reasonable description of reality near t_0.

Keeping the first three terms, and dividing by the current scale factor $a(t_0)$

$$\frac{a(t)}{a(t_0)} \approx 1 + \left.\frac{\dot{a}}{a}\right|_{t_0} (t - t_0) + \frac{1}{2}\left.\frac{\ddot{a}}{a}\right|_{t_0} (t - t_0)^2 \qquad (6\text{-}2)$$

as we define $a(t_0) = 1$, and $H_0 \equiv \left.\frac{\dot{a}}{a}\right|_{t_0}$, we have

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 \tag{6.3}$$

where q_0 is the "deceleration parameter"

$$q_0 \equiv -\left(\frac{\ddot{a}\, a}{\dot{a}^2}\right)_{t_0} = -\left(\frac{\ddot{a}}{a H^2}\right)_{t_0} \tag{6.4}$$

the negative sign is historical, from the 1950s when we thought the expansion was decelerating ($q_0 > 0 \rightarrow \ddot{a} < 0$).

- we can now relate q_0 to our model of the Universe using the acceleration equation (4.25), written as a sum over N energy density components

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} \sum_{i=1}^{N} \varepsilon_i (1 + 3w_i) \tag{6.5}$$

dividing by H_2 and changing sign

$$\frac{-\ddot{a}}{a H^2} = \frac{1}{2} \frac{8\pi G}{3c^2 H^2} \sum_{i=1}^{N} \varepsilon_i (1 + 3w_i) \tag{6.6}$$

and because $\varepsilon_c = 3c^2 H^2 / 8\pi G$, we can take this equation at t_0 to get

$$q_0 = \frac{1}{2} \sum_{i=0}^{N} \Omega_{i,0} (1 + 3w_i) \tag{6.7}$$

which in a universe with matter, radiation, and Λ, is

$$q_0 = \Omega_{r,0} + \frac{1}{2} \Omega_{m,0} - \Omega_{\Lambda,0} \tag{6.8}$$

thus, if we can measure q_0, we get a joint constraint on the density parameters. As $\Omega_{r,0}$ is small (i.e. we are currently dominated by matter and Λ, this constraint in on $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$.

- A matter dominated universe has $a(t) \propto t^{2/3}$, so $\ddot{a} < 0$ and $q_0 > 0$ (i.e. decelerating). A cosmological constant universe has $a(t) \propto e^t$, so $\ddot{a} > 0$ and $q_0 < 0$ (i.e accelerating).

- Assuming a flat universe, we can relate luminosity distance (i.e. something we can measure) to q_0 (and H_0)

$$d_L \approx \frac{c}{H_0} z \left(1 + \frac{1 - q_0}{2} z\right) \qquad (69)$$

(see Ryden section 6.2). Thus, if we can get d_L for objects at a range of z, we can measure H_0 and q_0, and constrain the density parameters $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$.
↳ (1st term - linear expansion, 2nd term - any acceleration/deceleration - expansion during photon travel)

Standard candles

- Measuring the luminosity distance requires us to know the intrinsic luminosity L. To use equation (6.9) we also need flux f and redshift z; these are easy, but knowing L is not

  - we need a class of astronomical objects whose luminosity is known, or can be predicted based on other properties. To do this at redshift ~ 1 requires very ($\sim 10^9 L_\odot$) luminous objects. Such objects are called "standard candles".

- Cepheids

  - these are "supergiant" stars, with mean luminosities $\bar{L} \sim 400$ to $40{,}000\ L_\odot$. Their luminosity is variable due to pulsations. The key to their use as standard candles was discovered by Henrietta Leavitt, who found that the average flux was correlated with the period for stars in the Small Magellanic Cloud (i.e. stars that are all at the same distance). The period-luminosity relation means that by measuring the period, L can be predicted, and d_L estimated.

  - This was one of the reasons for launching the Hubble Space Telescope, but still only works out to distances of about 30 Mpc. To go farther requires more luminous objects.

- Type Ia supernovae

  - these are very luminous (3 to $5 \times 10^9\ L_\odot$) events that occur when a white dwarf star exceeds about $1.4 M_\odot$ (the Chandrasekhar mass). This generally happens as the white dwarf is accreting material from a stellar companion. Above this mass the white dwarf gravity overcomes the electron degeneracy pressure that supports the star, and it collapses. The increased density triggers runaway nuclear fusion and the result is a very luminous explosion.

- just as Cepheids have a period-luminosity relation, type Ia supernovae have a shape-luminosity relation; the rise and fall of the flux over time is a good predictor of the peak luminosity.

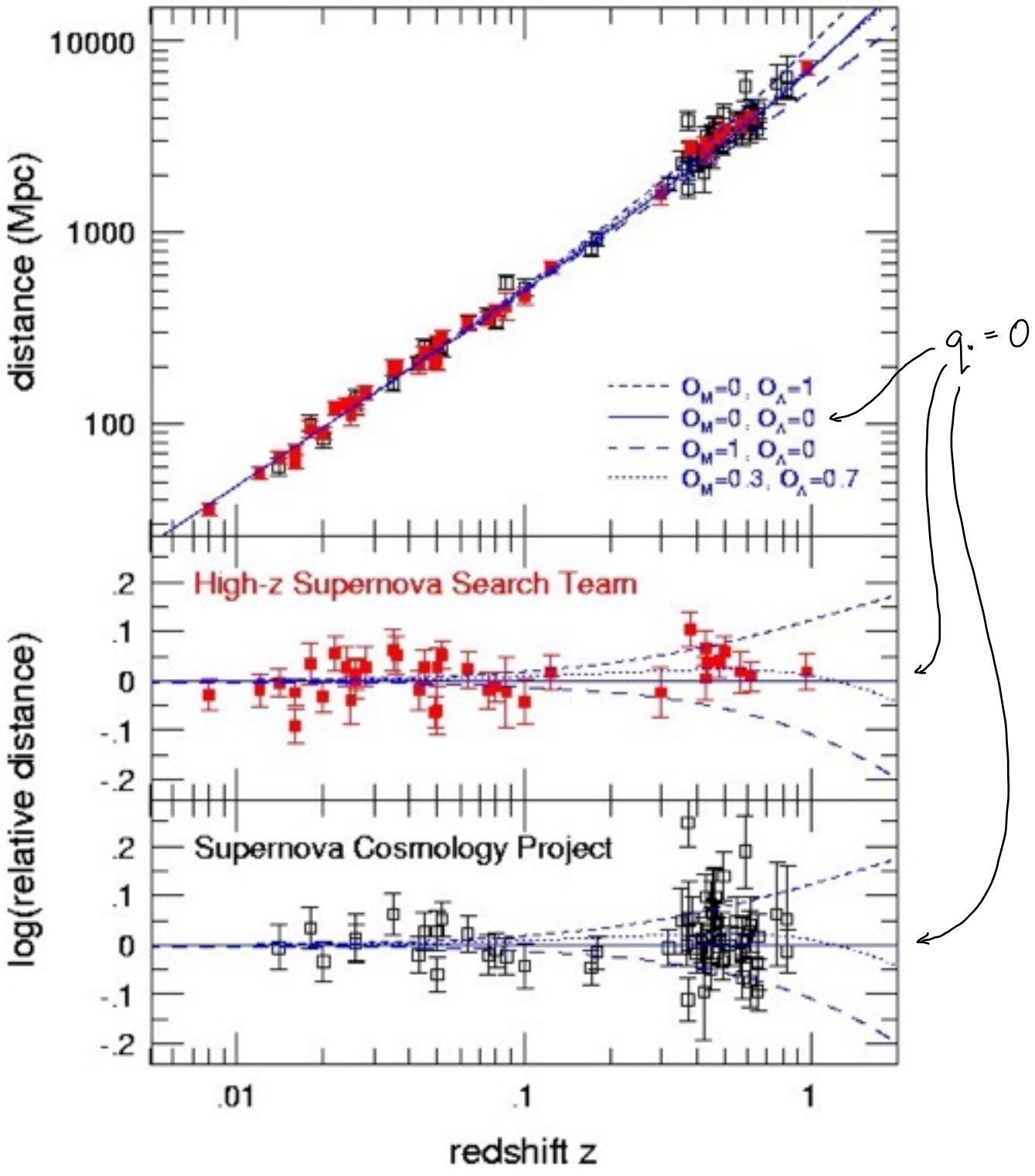
Measuring expansion and acceleration

- Now all we need to do is obtain distances and redshifts for a sample of standard candles. This is a major task that was first successfully carried out for high redshifts ($z < 1$) in the 1990s.

  - main result was that the supernovae appeared a bit fainter (i.e. more distant) than the should in a flat matter dominated universe, implying that the Universe's expansion is accelerating (and therefore that $\Omega_{\Lambda,0} > 0$).

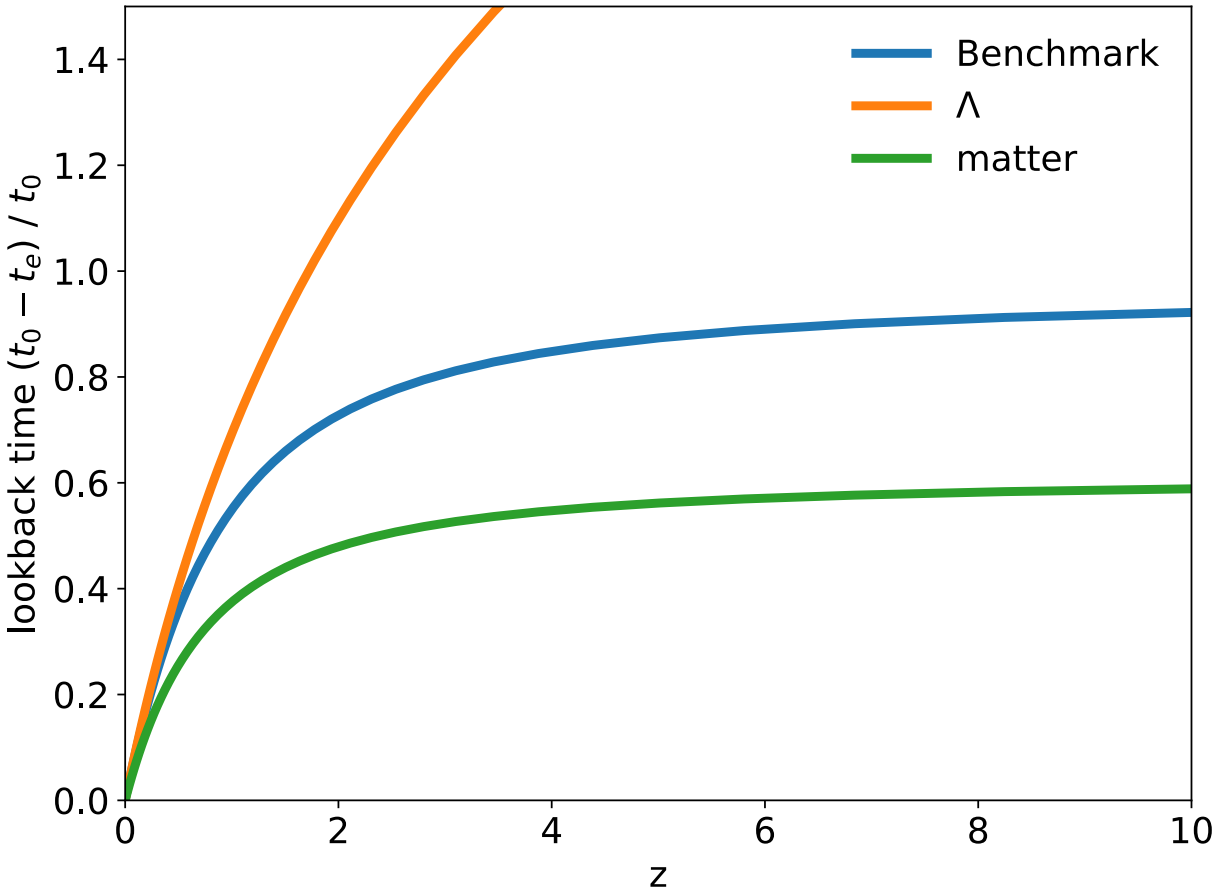
    the reason is that the redshift tells us the scale parameter at the time the supernova exploded. Relative to a universe expanding at a constant rate H_0, a universe whose expansion was slower in the past takes longer to reach the current scale factor, meaning that the light has travelled for longer and over a greater distance, and thus has fainter supernovae at a given z.

  - at low z this method yields a measurement of H_0, by fitting a straight line to cz vs. d_L



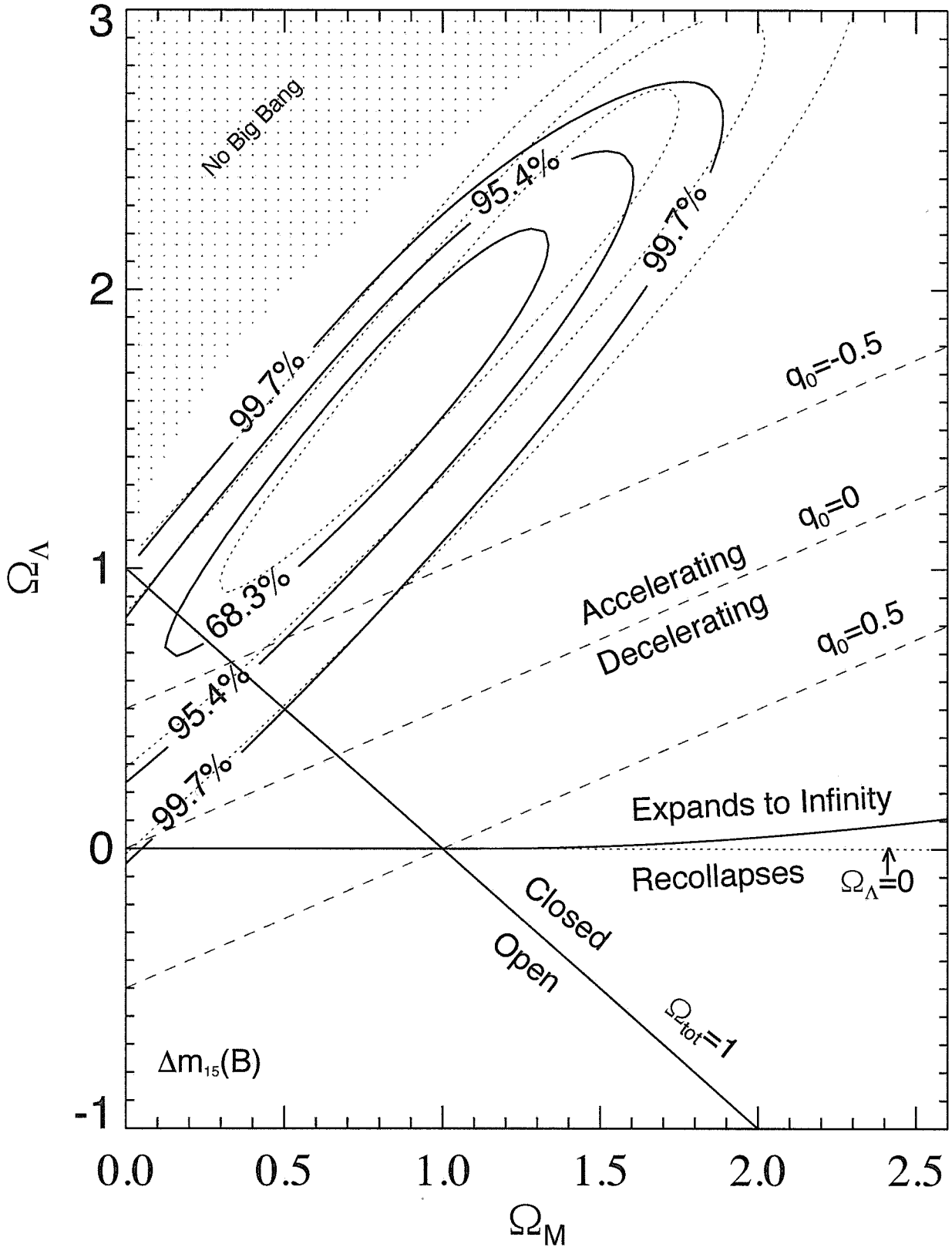
Evidence for an accelerating Universe and a cosmological constant

Leibundgut & Sollerman, 2001, Europhysics News, 32, 4,
using data from Riess et al. (1998) and Perlmutter et al. (1998)



Lookback time for flat universes (i.e. time since photon emission at a given z).

A Lambda-dominated universe was expanding more slowly in the past (compared to the benchmark), so for a given z (or scale factor) the evolution takes longer to reach the current scale factor. Consequently light emitted at a given z has travelled a greater distance, and appears fainter, in universes with greater acceleration.



Joint constraints on the matter and cosmological constant density parameters from type Ia supernovae

Riess et al. 1998, AJ, 116, 1009.
This paper has been cited ~11,100 times, and lead to a share of the Nobel prize